

1 **A Robust Optimization Approach for Selecting Urban Fire Engine Company Locations**

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5 **Abstract**

6 When a fire breaks out in a city, an emergency call to 911 is made and a fire engine
7 company responds to the incident. This average response time is an important system met-
8 ric that must be kept under a suitable threshold. The goal of this project is to study the re-
9 lationship between the number and locations of fire engine companies and their response
10 times to help cities optimize resources. Since the number and locations of fires are not
11 known apriori, any selected set of engine company locations must be robust across a wide
12 spectrum of fire incidents. Furthermore, if too much emphasis is placed on optimizing
13 resources and system costs, some engine companies could bear a disproportionate fraction
14 of the workload, leading to dissatisfaction or fatigue among firefighters. This project there-
15 fore also considers the incremental cost of ensuring equitable engine company workloads.
16 A methodology combining integer programming techniques, ensemble learning, and local
17 search using genetic algorithms is proposed to determine robust locations for the fire en-
18 gine companies. Tested with a data set for the city of Philadelphia, the results support the
19 hypothesis that optimizing fire engine company locations can result in significant savings
20 for resource-strapped cities.

21 **1. Introduction**

22 On July 5, 2014, four children were killed in a 3-alarm fire that broke out in
23 Philadelphia [14] and a similar fire entailed evacuating a hundred residents from an apart-
24 ment building more recently [17]. When a fire or medical emergency occurs, a call is
25 placed to 911 and even a minute reduction in the response time (i.e., the time between
26 the call and arrival of Emergency Medical Services (EMS) personnel at the site) could save
27 several lives [18]. A recent audit report published by the Office of the Controller for the
28 City of Philadelphia, reports that the National Fire Prevention Association (NFPA) has set
29 a standard where the response time for the fire engines should be within 5 minutes and
30 20 seconds after the call is dispatched for 90 percent of their runs. The Philadelphia Fire
31 Department responds to approximately 54,000 calls a year [18]. While response times can
32 be reduced by adding more ambulance shelters or fire engine company locations, cities are
33 already budget-constrained and must do their best with very limited resources.

34 The fields of operations research/management science have contributed immensely
35 to improving urban public services [8]. While EMS encompasses many domains, such as
36 locating ambulance shelters [1, 5], and optimizing police patrols [12], this research will
37 focus on studying the relationship between number and locations of fire engine companies
38 and their response times to emergency incidents.

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39 Previous work has studied the relationship between travel distances and travel
40 times for fire engines, and has also outlined an algorithm for engine company reloca-
41 tion(i.e., when an engine company is called away to respond to a fire, another engine com-
42 pany may be relocated to its base as a surrogate, in case a fire breaks out), but has ignored
43 the development of robust solutions for fire engine company location problems [10, 11].
44 We focus on obtaining robust solutions for fire engine company location problems. A so-
45 lution to a facility location problem is considered *robust* if it performs well under a wide
46 range of changes in any stochastic parameters associated with the system. Baron et al. [2]
47 consider robust facility location when demand for the product produced by the facility
48 varies considerably over multiple time periods. Cui et al. [7] treat the case of facility dis-
49 ruption (e.g., a factory producing parts for a car unexpectedly goes out of commission) and
50 develop robust solutions for this problem. Expanding on previous investigations, this pa-
51 per finds optimal solutions to engine company location selection under *spatial uncertainty*
52 *in demand* (i.e. it is not known precisely where fires will break out and engine companies
53 must be located to accommodate a wide range of scenarios for fire locations).

54 This paper addresses the following research questions:

- 55 1. What is the minimum number of fire engine companies needed to provide satisfac-
56 tory coverage of fires in a given city?
- 57 2. Given a required number of engine companies, where should these engine companies
58 be located?
- 59 3. What resources should be made available at each fire engine company? In addition
60 to the standard pumper-type truck, fire companies utilize a more functional and ex-
61 pensive ladder truck. How many *pumper* and *ladder* trucks should be placed at each
62 location?

63 To address these research questions, we propose a **decision support system** (DSS)
64 to determine engine company locations as outlined below and in Figure 1.

- 65 1. This paper proposes a new metric for measuring robustness in the context of fire
66 engine company location. Robustness is defined in terms of a tuple (β, p) , where at
67 least β percent of fires that break out in a fixed time period are covered with threshold
68 probability p (the probability of coverage p decreases with response distance (time)
69 traveled). More specifically, the research answers the following question: what is
70 the *minimum number of fire engine companies* needed within a city so that β % of fire
71 incidents can be covered with a *threshold coverage probability* p ?
- 72 2. Fire scenarios are generated via a Spatial Poisson process; the minimum number of
73 engine companies and their locations are identified via the integer program **EC-PSCP**
74 (*Equity Constrained Probabilistic Set Covering Problem*). This is Phase I in Figure 1.
- 75 3. The engine company locations identified for each scenario are combined via an **En-**
76 **semble Learning** algorithm, using the notion of “voting” for the most effective facil-
77 ities (Phase II in Figure 1).

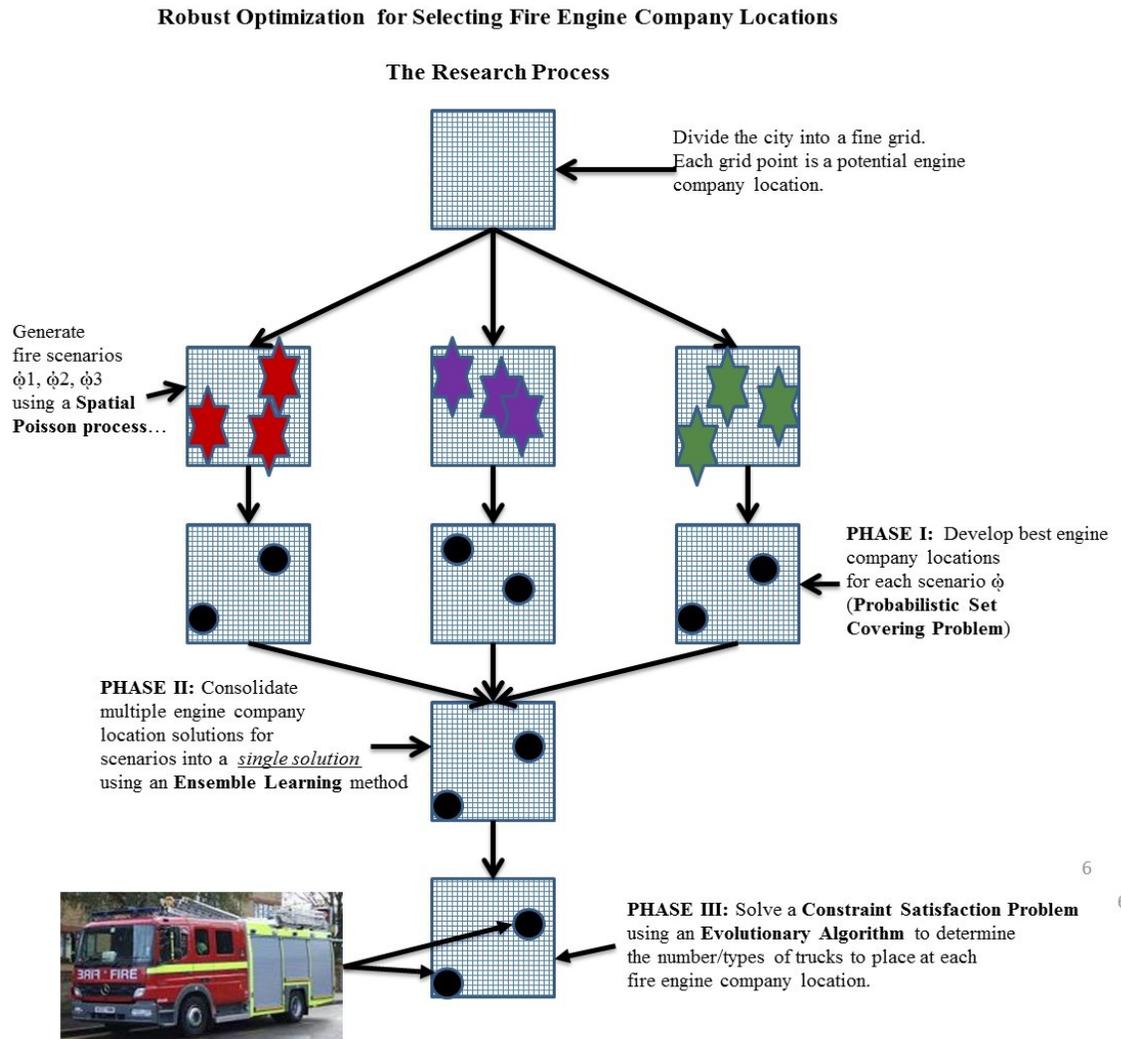
78 4. The current paper also considers *resource allocation* at each location. A variety of
79 resource configurations may be deemed feasible at this stage, in the spirit of “*solution*
80 *plurality*” [9]. Resources (e.g., pumper vs. ladder trucks) are allocated to each location
81 by solving a **Constraint Satisfaction Problem** using **Genetic Algorithms**.

82 *Section 2* of this paper describes the data used and provides details of the solution
83 algorithm. *Section 3* presents a discussion of computational results and *Section 4* concludes
84 the paper with relevant public-policy recommendations and offers future research direc-
85 tions.

86 **2. Solution Approach**

87 **2.1.A. Method Overview**

88 Figure 1 provides an overview of the procedure (algorithm) coded in Python,
89 which is illustrated using data obtained for the city of Philadelphia.



90

91

92 **Figure 1: Robust Optimization Process for Selecting Fire Engine Company Locations**

93

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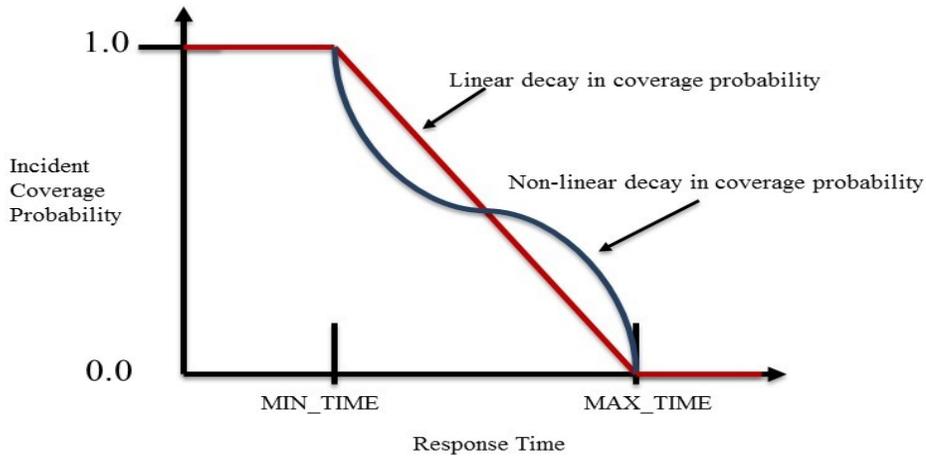
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94 The algorithm (solution methodology) requires *user-provided inputs* and creates
 95 certain *outputs*:

96 1. **User provided inputs:** The user must provide a tuple (β, p) , guiding the DSS to pro-
 97 vide solutions where β % of fires can be covered with probability p . The coverage
 98 probability itself depends upon the travel speed of fire engines and the distance be-
 99 tween the fire and the responding engine company. An expert user must provide
 100 a model for how the coverage probability varies with travel time (see Figure 2). In
 101 the computations presented in this paper, a linear decay in the coverage probabili-
 102 ty function is assumed. If a fire engine reaches a fire after t minutes, the coverage
 103 probability is stated below (t_{\min} and t_{\max} are user-specified parameters, see Figure
 104 2):

$$p(t) = \begin{cases} 1, & t \leq t_{\min} \\ -\frac{t-t_{\min}}{t_{\max}-t_{\min}} + 1, & t_{\min} < t \leq t_{\max} \\ 0, & t > t_{\max} \end{cases}$$

105 Response times less than t_{\min} are deemed satisfactory and response times exceeding
 106 t_{\max} are unacceptable.



107
 108

109 **Figure 2: Illustration of Two Coverage Probability Functions**

110

111 **Outputs created by the solution methodology:** *Number of engine companies* needed, their
 112 *arcGIS coordinates* and the *number and types of fire engines* (pumper or ladder trucks) at each
 113 location.

114 This **solution method**, discussed below, consists of the five parts (refer to Figure
115 1).

116 **2.1.B. Data**

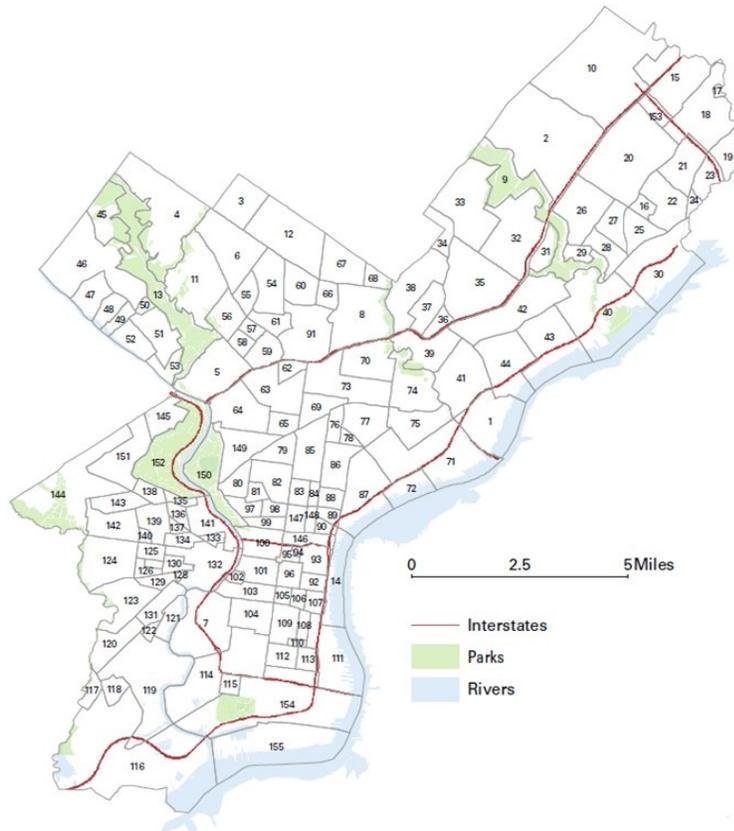
117 As a case study to illustrate the algorithm, we consider the city of Philadelphia.
118 The city is home to about 1.5 million residents and can be geographically partitioned into
119 155 neighborhoods (Figure 3). For each of these neighborhoods, a public-domain arcGIS
120 data set was obtained (see Figure 3) identifying the *neighborhood population* and the *neigh-*
121 *borhood centers* (i.e., the x and y coordinates for the centers in the arcGIS frame of refer-
122 ence). The precise spatial locations of fire incidents (for a 1-year period) were not available
123 for this project, but from public-domain information, it is known that a total of 54485 fire
124 incidents occurred during 2015 [18].

125 **2.2 Methods**

126 **2.2.1 Generating Candidate Engine Company Locations**

127 The city of Philadelphia consists of 155 neighborhoods (as shown in Figure 3
128 below) and for each neighborhood, a square grid is drawn, with the center of the square
129 aligning with the center of the neighborhood, thereby masking the whole city (see top box
130 in Figure 1). Each corner point of the grid is assumed to be a potential engine company
131 location and the mesh size for the grid is a parameter that can be controlled by the user. The
132 finer the mesh size, the closer the location problem is to a “continuous location” problem.

Philadelphia Neighborhoods



133

134

135

Figure 3: Map of Philadelphia Neighborhoods [16]

136 2.2.2 Generating Fire Location Scenarios

137 A scenario is simply a set of locations for fires that occur during a fixed period.
138 If a data set were available with the exact spatial locations of fires for a year, the contin-
139 uous spatial distribution to approximate the discrete set of real world fire events (the ob-
140 served data) can be fit using Kernel Density Estimation (KDE), which is a non-parametric
141 approach to density estimation from a set of discrete points [6, 13, 4]. The smoothing
142 parameter (bandwidth) determines the extent of spread of each point. Once the spatial
143 distribution is known, fire scenarios can be generated by sampling from this spatial distribu-
144 tion. However, fitting a spatial distribution for fire incidents is not the main objective of
145 this paper. Moreover, fire incident location data were unavailable for this project, although
146 an aggregate number for the total number of fires in the city of Philadelphia was available.
147 The total number of fires was allocated to each of the 155 neighborhoods in proportion

148 to the neighborhood's population. Fires were then randomly generated for each neighbor-
149 hood. One complete fire scenario consists of a set of spatial locations for fires in each of
150 the 155 neighborhoods. The scenario generation method repeatedly creates fire scenarios
151 for the city of Philadelphia. The next step is to solve for the best engine company locations
152 for each scenario using the method outlined in the next sub-section (2.3).

153 2.3 Solving the Equity Constrained Probabilistic Set Covering Problem (EC-PSCP) for 154 each Scenario

155 The Equity Constrained Probabilistic Set Covering Problem (EC-PSCP) repre-
156 sents the core of the methodology presented in this paper. An EC-PCSP is solved once
157 for each generated fire scenario (and the solutions later aggregated via an Ensemble Algo-
158 rithm (see section 2.4)). The EC-PCSP is an integer program that is a variant of the classical
159 set covering problem. As a preamble to solving the EC-PCSP, a set covering matrix A must
160 be developed. The matrix A has one row for each fire incident and one column for each
161 potential engine company location. A matrix entry a_{ij} is 1 if an engine company at location
162 j can cover a fire at i . The following steps enable the computation of matrix A .

163 **Step 1:** The Euclidean distance between each *candidate facility* j (node of the grid) and each
164 fire incident location i in the scenario is computed.

165 **Step 2:** The Euclidean distance is converted to a travel distance using a **DETOUR INDEX**
166 (equal to the ratio of actual travel to Euclidean distance) of 1.42. Prior research supports
167 the use of such an index [9, 3].

168 **Step 3:** Using the user-specified travel speed for fire engines, the travel time matrix T is
169 computed. T has (i, j) th entry = $t(i, j) = (\text{travel distance between } i \text{ and } j)/(\text{engine speed})$.

170 **Step 4:** Using the travel time, compute the *coverage probability* p (as in Figure 3) and deter-
171 mine if a facility at j can cover a fire at i . Develop a *set covering matrix* A , with (i, j) th entry
172 $a_{ij} = 1$ if the coverage probability \geq user-specified threshold probability and 0 otherwise
173 [Note: The user-specified threshold probability is the second field in the tuple (β, p)].

174 **Step 5:** Use the set covering matrix to develop a mathematical programming formulation
175 for the fire engine company location problem. The set covering formulation is solved opti-
176 mally by using the Gurobi optimization solver [15].

177 An example set covering matrix and the formulation for the Equity Constrained Proba-
178 bilistic Set Covering Problem **EC-PSCP** is explained below.

179 In this example scenario, five fires must be covered by a set of four engine com-
180 panies. The set covering matrix for this scenario is presented in Table 1.

181

182 **Table 1: The set covering matrix for example scenario, where columns denote EC =**
 183 **Engine Company, and rows are fire incidents**

	(EC 1)	(EC 2)	(EC 3)	(EC 4)
fire 1	1	1	1	0
fire 2	1	1	1	0
184 fire 3	1	1	1	1
fire 4	0	0	1	0
185 fire 5	0	0	1	1

186 The set covering matrix A has entry $a_{ij} = 1$ if an engine company at location (column) j can
 187 reach a fire at location (row) i with required coverage probability p . The constraint set is
 188 illustrated below for this example.

189 **The Equity Constrained Probabilistic Set Covering Problem (EC-PSCP):**

- 190 1. The mathematical program has a **variable** y_j for each node (*potential* EC facility).
- 191 2. y_j is 1 if an engine company is located at node j , and 0 otherwise (a switch variable).
- 192 3. The math program has a binary variable X_{ij} which is set equal to 1 if fire i is covered
 193 by an engine company located at j .
- 194 4. The math program has a switch variable S_i for each fire i . If $S_i = 1$ in the solution,
 195 fire i is NOT covered within the stipulated response time. The variable S_i provides
 196 a “pass” that allows the math program to pass on covering fire i (In this model, only
 197 β % of fires are covered within the stipulated response time).
- 198 5. A *scenario* is defined by a set of locations for fire incidents, drawn from a spatial
 199 distribution.
6. The mathematical program has five sets of constraints (explained below) in the *scenario* ω .
7. Every new *scenario* ω will lead to a new mathematical program because the locations
 of the fires will change.

200

201 **OBJECTIVE: Minimize** $\sum_{j=1}^{|J|} y_j$

202 **Illustration for Example: OBJECTIVE: Minimize** $\sum_{j=1}^4 y_j = y_1 + y_2 + y_3 + y_4$

203 **SUBJECT TO FIVE SETS OF CONSTRAINTS:**

204 **Constraint set I (cannot set X_{ij} to 1 unless y_j is also 1):**

$$\sum_i a_{ij} X_{ij} \leq |I| y_j \text{ for all engine companies } j \in J$$

205 *Illustration of Constraint Set I for Example:*

206 $|I| = 5$, since there are 5 fires in the example matrix.

$$X_{11} + X_{21} + X_{31} \leq 5y_1$$

207

$$X_{12} + X_{22} + X_{32} \leq 5y_2$$

208

$$X_{13} + X_{23} + X_{33} + X_{43} + X_{53} \leq 5y_3$$

209

$$X_{34} + X_{54} \leq 5y_4$$

210

211 **Constraint set II (either a fire i must be covered by an engine company y_j , OR the**
212 **“pass” variable S_i must be set equal to 1):**

$$\sum_j a_{ij} y_j \geq [1 - S_i] \text{ for all fires } i \in I$$

213

$$y_1 + y_2 + y_3 \geq [1 - S_1] \text{ (i.e., cover fire 1 if } S_1 = 0)$$

214

$$y_1 + y_2 + y_3 \geq [1 - S_2] \text{ (i.e., cover fire 2 if } S_2 = 0)$$

215

$$y_1 + y_2 + y_3 + y_4 \geq [1 - S_3] \text{ (i.e., cover fire 3 if } S_3 = 0)$$

216

$$y_3 \geq [1 - S_4] \text{ (i.e., cover fire 4 if } S_4 = 0)$$

217

$$y_3 + y_4 \geq [1 - S_5] \text{ (i.e., cover fire 5 if } S_5 = 0)$$

218

219 **Constraint set III: Enforce the Equity Spread Constraint**

$$\sum_i a_{ij} X_{ij} \leq \text{Max for all engine companies } j$$

220

$$\sum_i a_{ij} X_{ij} \geq \text{Min for all engine companies } j$$

221

$\text{Max} - \text{Min} \leq \text{EquitySpread}$ [this is a parameter provided by user]

222

Illustration of Equity Spread constraints for example :

223

$$X_{11} + X_{21} + X_{31} \leq \text{Max}$$

224

$$X_{12} + X_{22} + X_{32} \leq Max$$

225

$$X_{13} + X_{23} + X_{33} + X_{43} + X_{53} \leq Max$$

226

$$X_{34} + X_{54} \leq Max$$

227

$$X_{11} + X_{21} + X_{31} \geq Min$$

228

$$X_{12} + X_{22} + X_{32} \geq Min$$

229

$$X_{13} + X_{23} + X_{33} + X_{43} + X_{53} \geq Min$$

230

$$X_{34} + X_{54} \geq Min$$

231

$$Max - Min \leq EquitySpread$$

232 **Constraint set IV: At least β % of fires must be covered.**

$$\sum_{i=1}^{|I|} S_i \leq [1 - \beta] \times |I|$$

233 *{equivalently, $\sum_{i=1}^{|I|} \sum_{j=1}^{|J|} X_{ij} \geq \beta \times |I|$ }*

234 Illustration of constraint set IV for example, where $\beta = 0.8$ is assumed.

235 $S_1 + S_2 + S_3 + S_4 + S_5 \leq \{(1 - 0.8) \times 5\} = 1$ (This ensures 4 out of 5 fires covered)

236 **Constraint Set V:**

237 **Declare all variables to be BINARY (0 or 1)**

$$X_{ij}, y_j, S_i \in \{0, 1\}$$

238 In the example:

239 y_1, y_2, y_3, y_4 are all binary (0 or 1) variables

240 $S_1..S_5$ are ALSO binary (0 or 1) variables

X_{ij} is binary for $i = 1, \dots, 5$ and $j = 1, \dots, 4$

241

242

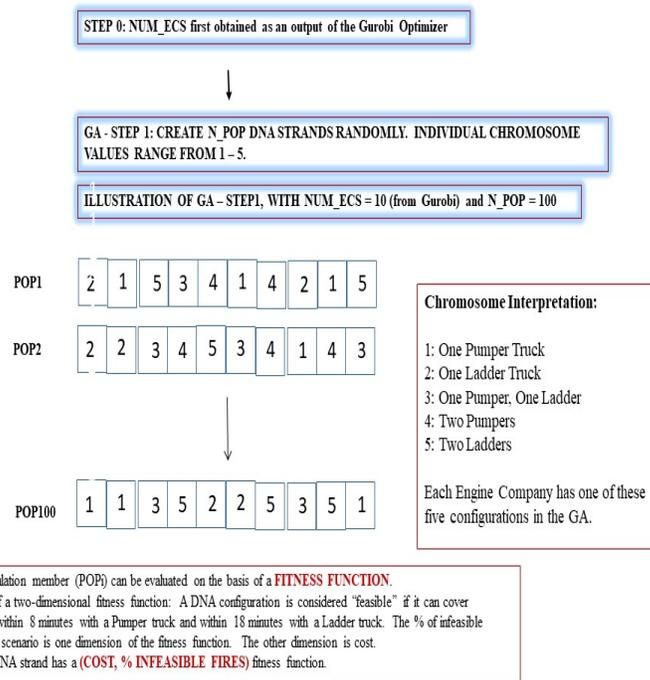
243 2.4 Combining Multiple Solutions with an Ensemble Algorithm

244 The Ensemble Algorithm combines the location solutions for each generated sce-
245 nario into a single location solution. Each candidate facility j (in *any* of the scenario solu-
246 tions) is given a cumulative weight that depends upon the distance of facility j to either a)
247 all fire scenarios *or* b) to all other selected engine company locations. Finally, all the en-
248 gine company locations in *any* scenario solution are rank-ordered based upon this weight.
249 Engine companies are successively selected from this rank-ordered list in a greedy fashion
250 (i.e., the next best facility is the one that covers the most number of uncovered fires) until
251 the robustness criterion of β % of fires covered is achieved.

252

253 2.5 Solving a Constraint Satisfaction Problem using a Genetic Algorithm

254 When the Ensemble Algorithm is completed, the number of engine companies
255 selected is known along with their precise arcGIS locations. The purpose of Phase III
256 (in Figure 1) is to place *resources* at these engine companies. Two kinds of resources are
257 considered, pumper engines (that cost about \$300,000) and ladder engines (that are more
258 expensive, upwards of \$900,000). The goal of the Genetic Algorithm is to determine the
259 *type* and *number* of engines at each location. A Genetic Algorithm approach was selected
260 as the search procedure as it offered a natural way to encode the solution space. At least
261 one engine must be placed at each location. In addition, two kinds of constraints must be
262 satisfied by a solution to the Constraint Satisfaction Problem: a) a constraint on the overall
263 cost of engines for the entire city (a budget constraint) and b) every fire must be responded
264 to within 8 minutes by some type of engine (pumper or ladder) and every fire must be
265 responded to with a ladder truck within 18 minutes (these parameters are used for illus-
266 trative purposes only and can be changed by the user of the DSS). The *fitness function* for
267 the GA is a two-component vector; one component is total cost, and the other is percentage
268 of fires not covered as per constraint b) above. A simple algorithm combining crossover and
269 mutation of genes (where a gene is a resource profile for *all* the selected EC locations) has
270 been designed for this phase. Figures 4 and 5 below provide schematic representations of
271 the Genetic Algorithm steps.



272

273

Figure 4: Schematic of Genetic Algorithm.

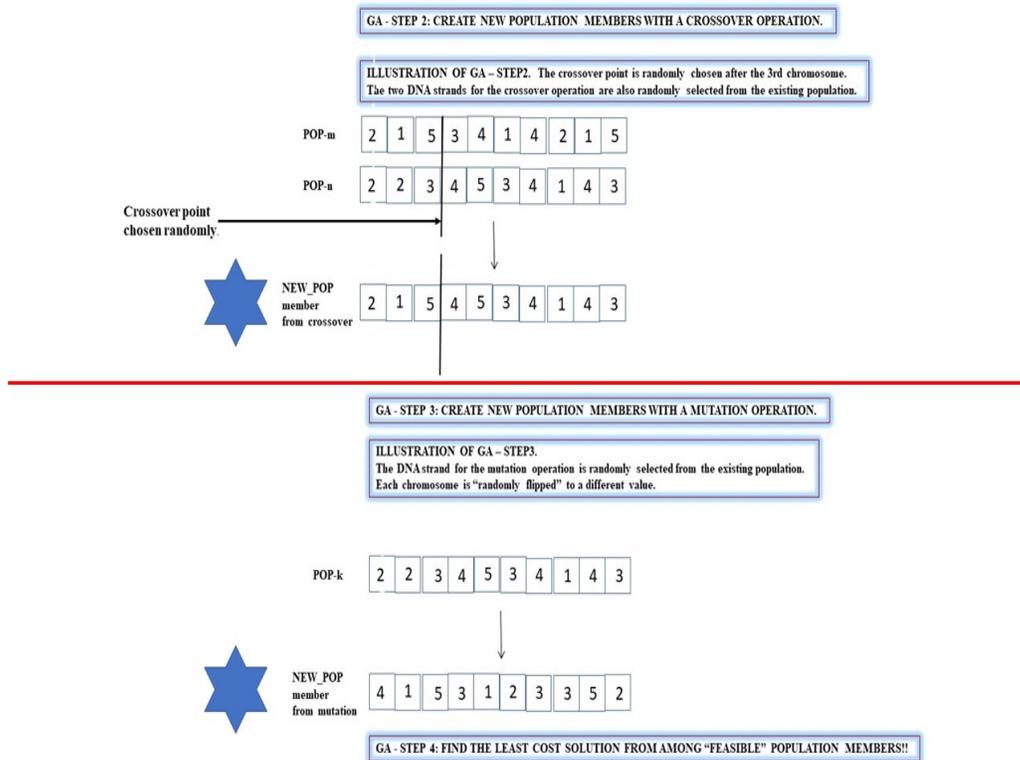
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276

277

Figure 4 provides a schematic representation of the Genetic Algorithm step. The Gurobi Optimizer [15] is used to solve the probabilistic set covering problem in Phase I. The optimizer determines the location of the Engine Companies. The Genetic Algorithm determines how many pumper and ladder trucks to place in each location.



278

279 **Figure 5: Schematic for Cross Over and Mutation Operations using Genetic Algorithms**

280 Figure 5 provides a schematic representation for cross over and mutation oper-
 281 ations in the Genetic Algorithm. There were 100 cross over operations followed by 100
 282 mutation operations for the computational results presented in this paper. However, the
 283 user has the ability to set parameters for the number of crossover and mutation opera-
 284 tions performed. While in principle, there could be genes that code for a larger number
 285 of resources at each Engine Company, in practice, budget limitations would make gene
 286 configurations other than the ones discussed in this paper unlikely.

287 **3. Discussion of Computational Results**

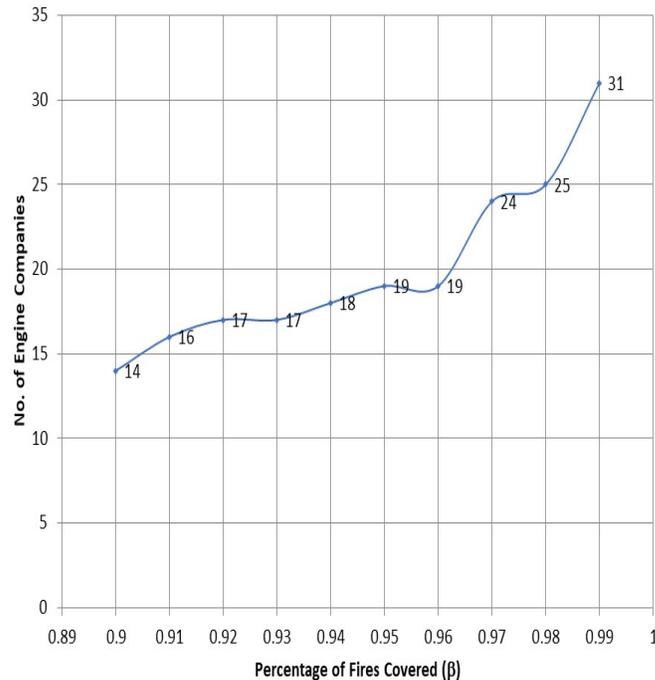
288 The methodology presented in the previous section was coded in the Python pro-
 289 gramming language. The Equity Constrained Probabilistic Set Covering Problem (EC-
 290 PSCP) was solved using the Gurobi optimization package [15]. All computations were per-
 291 formed on a 64-bit Lenovo laptop. In conformance with standard practice in the machine
 292 learning literature, five training scenarios were created to find engine company solutions
 293 and these solutions were validated using five test scenarios (results reported in this section

294 are generally averages for the five test scenarios). Please note, the user can also select the
295 number of training and test scenarios. The discussion of computational results is further
296 organized into the following sub-sections:

- 297 1. Impact of the robustness parameter β
- 298 2. Impact of engine speed
- 299 3. Solution quality in terms of engine company equity
- 300 4. Impact of coverage probability function assumptions
- 301 5. Illustrative performance of the genetic algorithm.

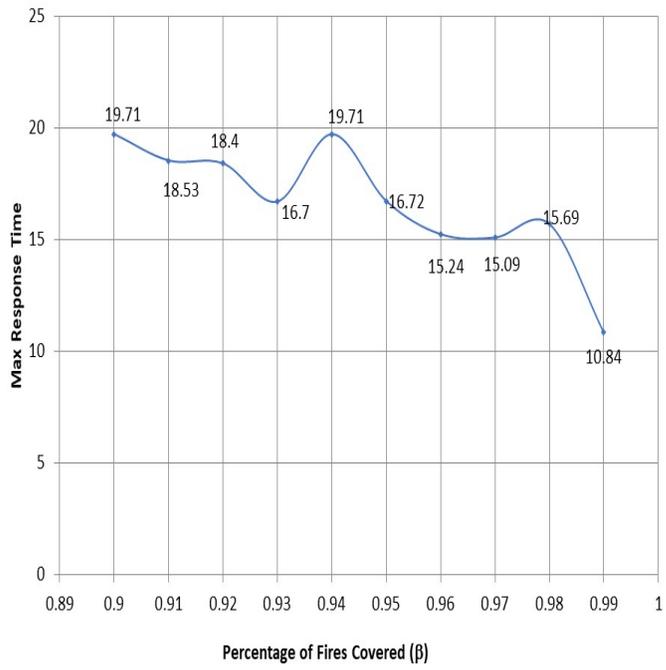
302 3.1 Impact of the Robustness Parameter β

303 The parameter β , which is the proportion of fires covered effectively in a solution
304 is the fundamental measure of “robustness” of any engine company solution. The higher
305 the β , the more engine companies will be required. The following charts demonstrate the
306 impact of β on the number of engine companies required and the maximum response time.



307

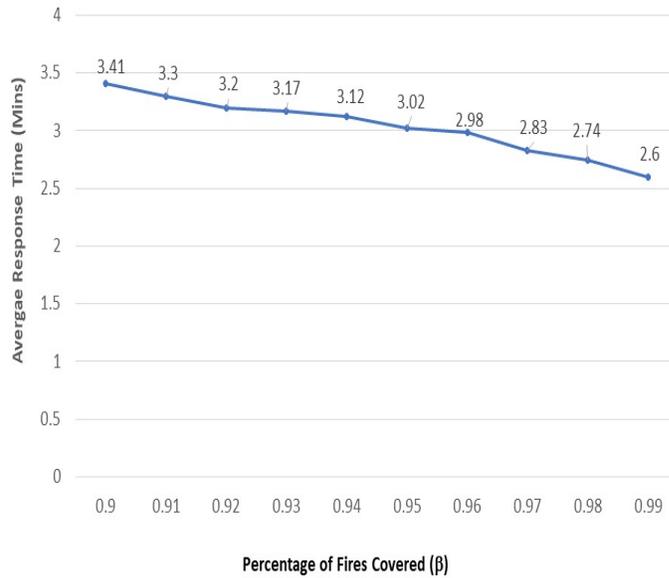
308 **Chart 1: Impact of β on the Number of Engine Companies Required**



309

310

Chart 2: Impact of β on Maximum Response Times



311

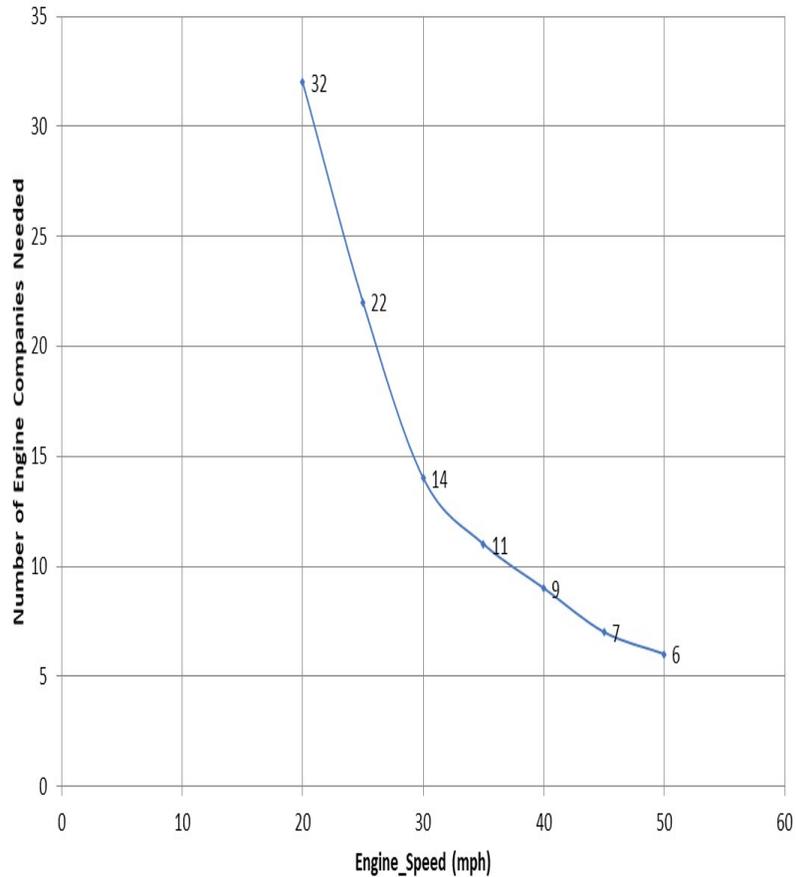
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Chart 3: Impact of β on Average Response Times

313 As β increases from 0.9 to 0.99, the average response time decreased from 3.41
 314 to 2.6 minutes. However Charts 1, 2, and 3 together document the price of robustness (#
 315 engine companies required more than doubles as β increases from 0.9 to 0.99) and the ben-
 316 efit of robustness (the maximum response time for *any* fire almost drops by 10 minutes).
 317 Policy makers must thoughtfully choose an operating point from these results.

318 **3.2 Impact of Fire Engine Travel Speed**

319 The speed of travel for any incident may depend upon extraneous factors such as
 320 time of day and traffic conditions. However, most cities have an inherently latent capacity
 321 for allowing fire trucks to move about the city with a certain speed. Chart 4, below, shows
 322 a steep increase in the number of engine companies is required for travel speeds less than
 323 30 mph. In this research, rather than view the fire engine travel speed as a given of the
 324 environment, it is assumed that policy makers have a limited ability to influence the speed
 325 of travel for emergency response (e.g., by levying extreme fines for obstructors).



326

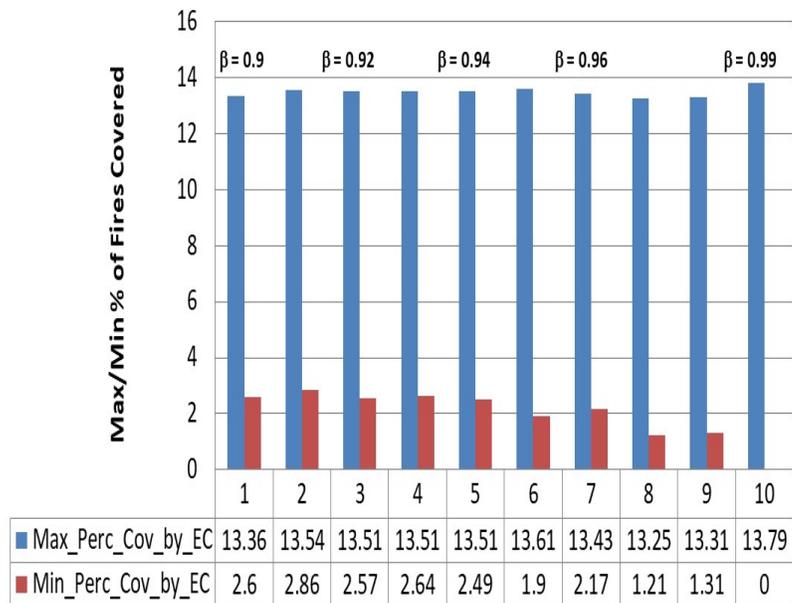
327 **Chart 4: Impact of Engine Speed on Number of Engine Companies Required**

328

329 **3.3 Equity for Engine Company Workloads**

330 A good solution for the engine company location problem must pay attention to
 331 the workloads allocated to various fire engine companies (some companies cannot remain
 332 idle too much of the time). Equity considerations can be explicitly added to the EC-PSCP,
 333 but this also makes the problem more difficult to solve. Future research should also ex-
 334 plore improving the equity with Phase III (local search for better solutions). Chart 5 below
 335 indicates a discrepancy of about 10% (between the busiest and most idle engine compa-
 336 nies) when EC-PSCP is solved *without* any constraints on equitable workloads. Moreover,
 337 equity is harder to achieve for higher values of β .

338



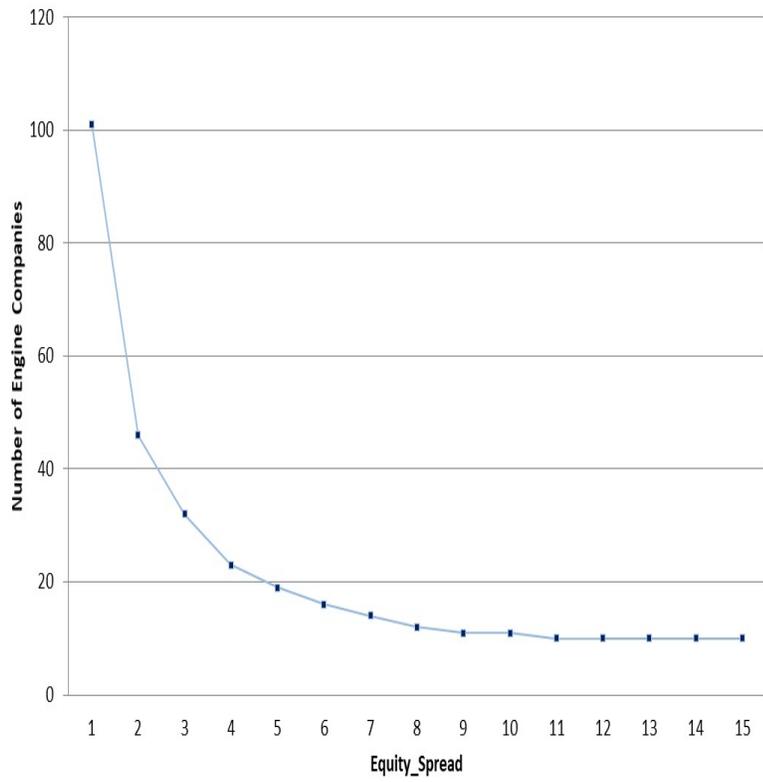
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340

341 **Chart 5: Impact of Equity Spread on Engine Company Workloads as β Increases**

342

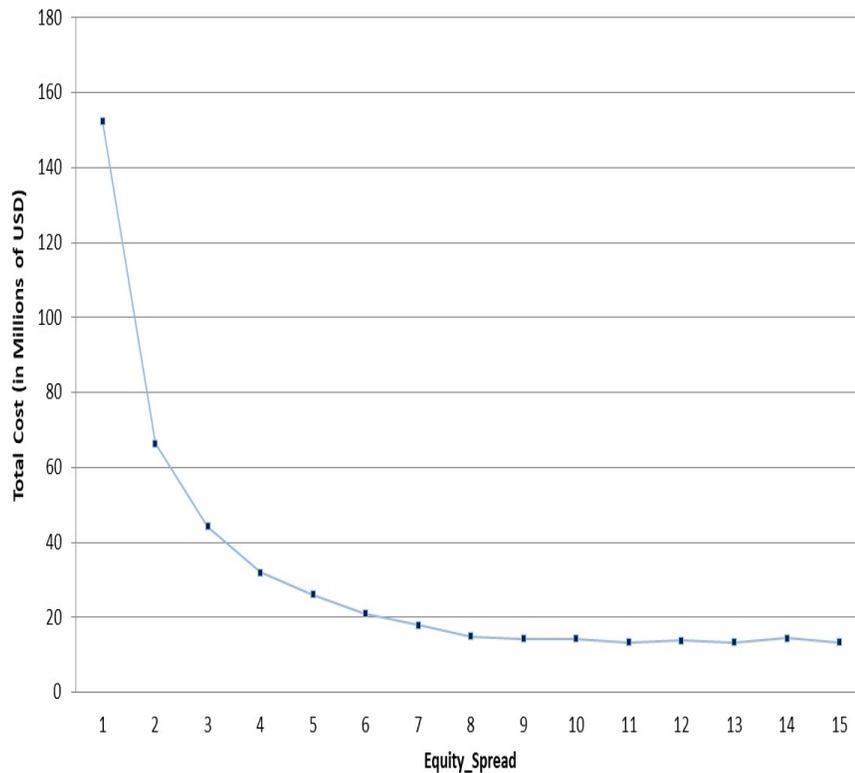
343 However, EC-PSCP can also be solved parametrically by varying the “Equity
 344 Spread” limit in the formulation. Clearly, the number of engine companies needed, as well
 345 as the total system cost, can be expected to increase as the Equity Spread limit decreases.
 346 The behavior of the system with decreasing Equity Spread is characterized in Charts 6 and
 347 7 below.



348

349

Chart 6: Impact of Equity Spread on Number of Engine Companies Required



350

Chart 7: Impact of Equity Spread on Total Cost

351

352 For low levels of Equity Spreads, the system cost increases by as much as \$1M for
 353 each percentage point reduction in the Equity Spread. From these plots, it can be observed
 354 that the increase in system cost depends upon the current level of the equity spread. So
 355 a decrease in Equity Spread from 15 to 14 has a much smaller impact on system cost as
 356 compared to a decrease from 2 to 1.

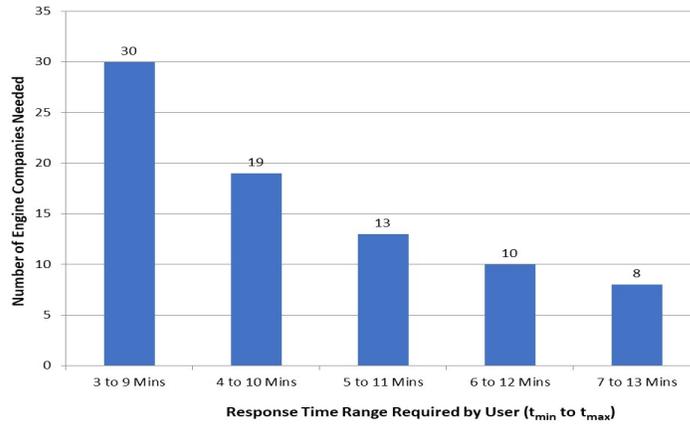
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3.4 Impact of Coverage Probability Function Assumptions

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359 The user-specified coverage probability function of Chart 1 is a key determinant
 360 of the solution quality. In particular, t_{\min} and t_{\max} in Chart 1 influence the form of the cov-
 361 erage probability function and in turn the number of engine companies needed. In some
 362 cases, the coverage probability function may be the subjective opinion of an expert user of
 363 the DSS. For this reason, the impact of changing the form of the coverage probability func-
 364 tion is studied below in Chart 8. The National Fire Protection Association (NFPA) standard

365 calls for the first due fire engines to arrive on scene within 5 minutes and 20 seconds after
366 being dispatched for 90 percent of their runs. As expected, **response times lower than**
367 **about 6 minutes impose an enormous cost on the system.** [18]



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369 **Chart 8: Impact of Response Time Standards on Number of Engine Companies**

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3.5 Illustrative Performance of the Genetic Algorithm (GA)

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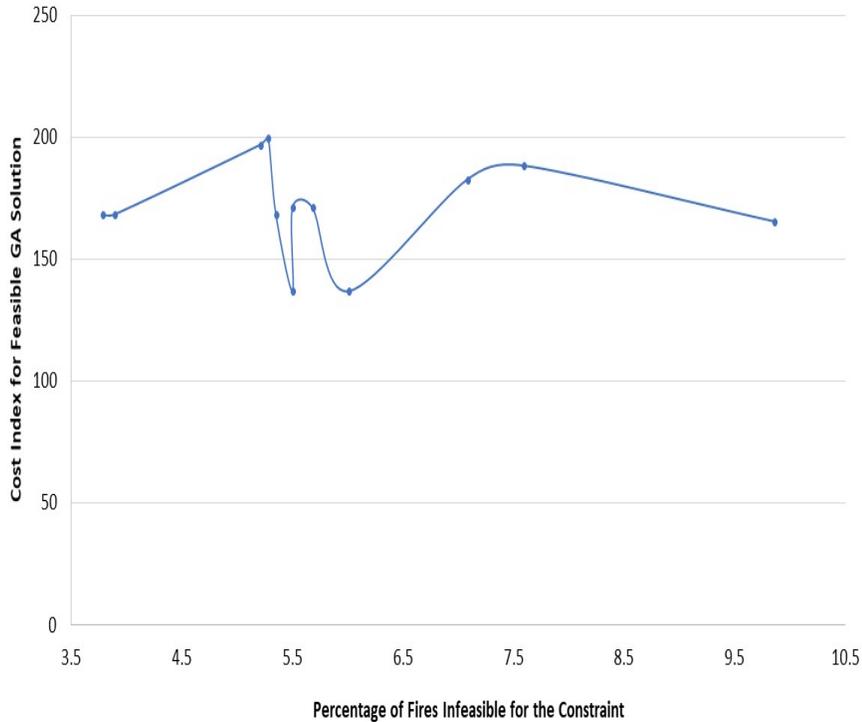
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Phase III of the solution procedure employs a GA to place resources (pumper and ladder trucks) at each chosen engine company location. Only resource allocations are changed in this phase and engine company locations themselves are fixed. The GA uses a two-dimensional fitness function. The first dimension is the *budget* used. Given that there must be at least one engine at each location, a lower bound for the cost is the number of engine companies multiplied by the cost of a pumper (less expensive) truck. All resource configuration costs are expressed as an index with respect to this base cost (i.e., a cost of 150 means the resource configuration is 50% more expensive than the cost of placing one pumper truck at all stations). Likewise, the second component of the fitness function represents the *percentage of fires* that do not satisfy the constraint “first response (by any type of truck) within 8 minutes and a ladder truck available within 18 minutes”. The GA searched for a resource configuration with a cost index less than 200 and the percentage of infeasible fires less than 10% (For illustration only: the user can tune feasibility parameters for genes). Chart 9 below illustrates how these two GA fitness attributes trade off in the set of feasible solutions found. The computational experiments indicate that solutions feasible to these two constraints are hard to find (less than 10% of the population members generated by GA were “feasible”).



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391 **Chart 9: GA Performance: System Cost vs. Percentage of Fires Not Covered**

392 Finally, a check was made for overfitting. If the β calculated for the test data
 393 sets is significantly lower than the β stipulated for the training data sets, overfitting has
 394 occurred. It was validated that the model was not overfitting the data. There were some
 395 “negative” results in the computations however. For instance, in the Ensemble Algorithm,
 396 it did not matter whether facilities were combined based upon distance to fires in scenarios,
 397 or distance to other facilities chosen in other solutions. Moreover, the number of candidate
 398 engine company locations that the DSS started with did not matter beyond a point. Most
 399 of the reported computations started with 625 candidate engine company locations.

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4. Conclusions, Public Policy Implications, and Future Research Opportunities

4.1 Conclusions

402 This paper develops a robust optimization approach for locating fire engine com-
 403 panies. The main dimension of robustness addressed is the spatial uncertainty of fire in-
 404 cident locations. For this problem, the current paper provides a solution algorithm to find
 405 the minimum number of engine companies needed so that β % of fires can be covered with
 406

407 probability p . The main contribution is the development of a Probabilistic Set Covering
408 Model formulation for this problem. In addition to the EC-PSCP, the paper also develops
409 a method to combine solutions from different scenarios (the Ensemble algorithm) and a
410 local search procedure for placing resources at chosen fire engine company locations.

411 4.2 Public Policy Implications

412 The solution algorithm and associated computational results raise several impor-
413 tant public policy issues:

- 414 1. One of the responsibilities of city government is to choose an appropriate level of
415 coverage β . **In fact, the required number of engine companies more than doubles**
416 **as β increases from 90 to 99%.** Policy makers should carefully evaluate the trade-off
417 between the increased cost of opening additional locations and the real benefits from
418 covering the last few percentiles.
- 419 2. As engine speed increases, the number of engine companies required decreases (as
420 expected). The data also seem to indicate that EMS responders must try to achieve
421 an engine speed of at least 30 mph. **For engine speeds < 30 mph, there is again a**
422 **steep increase in the number of engine companies required.** Given that this pa-
423 rameter has the greatest impact on the number of engine companies needed, policy
424 makers should consider policy options that can increase fire engine travel speeds.
425 Options such as special lanes for fire engine travel (like what is done for mass transit
426 buses already) and higher fines on the roads for obstructing fire engine travel must
427 be considered.
- 428 3. Special attention must be given to **engine company workload equity.** The computa-
429 tional results indicate that there might be as much as a 10% difference in the propor-
430 tion of fires tackled by the busiest engine company and the engine company with the
431 smallest workload. Territory design for engine companies (to attain equitable work-
432 loads) is an important extension of this project, as disparity may cause difficulties
433 with labor unions or contract workers.
- 434 4. Finally, achieving response times of less than 5 minutes is extraordinarily difficult.
435 **There is a steep increase in the number of engine companies required for achiev-**
436 **ing average response times less than 5 minutes.** Policy makers should consider
437 options wherein a sufficient amount of education and equipment is provided locally
438 (e.g., fire extinguishers) at building sites, so that local residents can contain the im-
439 pact of the fire for about 6-8 minutes. If this 6-8-minute time threshold can be man-
440 aged locally, the city can also drastically lower the costs of opening more fire engine
441 companies.

442 4.3 Future Research Opportunities

443 The current research also has some modeling limitations. Travel times are com-
444 puted ignoring conditions like traffic or time of day. Some fires require multiple response
445 units and it may not be sufficient to dispatch just the closest unit. Some fires may also
446 require specific equipment such as ladder companies that may not be available at the clos-
447 est facility (this issue is partly addressed by the Genetic Algorithm). Fire engines may

448 also be unavailable due to external circumstances such as maintenance and maintenance
449 plans must be factored into developing engine company locations. In terms of model-
450 ing enhancements, this exercise can be repeated with GIS mapping tools to formulate the
451 same model at a more fine-grained level (e.g., include details of one-way streets, traffic
452 lights/intersections). Finally, the model developed herein has a rich set of other appli-
453 cations such as ambulance shelter location, police patrol improvement and logistics for
454 emergency response (e.g., for hurricanes such as Katrina), where spatial demand uncer-
455 tainty considerations are required.

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